# Co-ordinate Geometry

Coordinate geometry is used to represent algebraic relations on graphs.

We shall be dealing with two-dimensional problems, where there are two variables to be handled. The variables are normally denoted by the ordered pair (x, y).

The horizontal axis is the X -axis and the vertical axis is the Y-axis. If the coordinates of a point on the XY plane is (x, y), it implies that it is at a perpendicular distance of x from the Y-axis and at a perpendicular distance y from the X-axis. The point of intersection of the X and Y-axis is called the origin and the coordinates of this point is (0, 0).

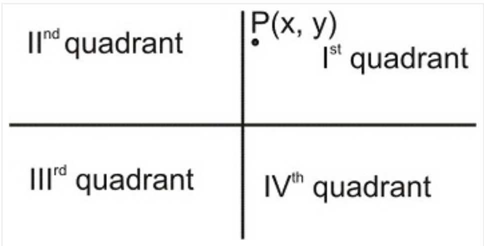
Signs of the coordinate ‘x’ and ‘y’ depends on the quadrant in which the point lies

I quadrant : x coordinate is positive; y coordinate is positive

II quadrant : x coordinate is negative; y coordinate is positive

III quadrant : x coordinate is negative: y coordinate is negative

IV quadrant : x coordinate is positive; y coordinate is negative

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**Some fundamental formulae:**

1. Distance between the points $(x_1, y_1)$ and $(x_2, y_2)$ is $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Slope:**

Angle made by the line with the positive direction of x - axis is called the inclination of the line.

If $\theta$ is the inclination, then ‘tan $\theta$’ denotes the slope of the line.

Slope of the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$ ; . The slope is also indicated by m.

**Slope intercept form:**

All straight lines can be written as y = mx + c, where m is the slope of the straight line, c is the Y intercept or the Y coordinate of the point at which the straight line cuts the Y-axis.

**Point slope form:**

The equation of a straight line passing through $(x_1, y_1)$ and having a slope m is $y - y_1 = m(x - x_1)$

**Two point form:**

The equation of a straight line passing through two points $(x_1, y_1)$ and $(x_2, y_2)$ is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

**Intercept form:**

The intercept form of a line is $\frac{x}{a} + \frac{y}{b} = 1$

Where ‘a’ is the intercept on x-axis and ‘b’ is the intercept on y-axis.

The point of intersection of any two lines of the form y = ax + b and y = cx + d is same as the solution arrived at when these two equations are solved.

**Perpendicular Distance:**

The length of perpendicular from a given point $(x_1, y_1)$ to a given line ax + by + c = 0 is $\left| \frac{ax_1 + by_1 + c}{\sqrt{(a^2 + b^2)}} \right| = p$ , where p is the length of perpendicular. In particular, the length of perpendicular from origin

**Distance Between two parallel lines:**

Distance between two parallel straight lines $ax+by+c_1 = 0$ and $ax+by+c_2 = 0$ is given by $\dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

**Actute Angle between two lines:**

If $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$ are equations of two lines, then acute angle (q) between them is given by $\text{Cos}\,\theta = \dfrac{|a_1 a_2 + b_1 b_2|}{\sqrt{(a^2{}_1 + b^2{}_1)(a^2{}_2 + b^2{}_2)}}$

If $m_1$ and $m_2$ are slopes of two straight lines, then acute angle $(\theta)$ between them is given by $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

**Area of the Triangle:**

The area of a triangle whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ =

$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

**Solved Examples:**

1. What is the distance between points A (4, 3) and B (7, 7)

Solution:

$(x_1, y_1)$ = (4, 3); $(x_2, y_2)$ = (7, 7)

Distance between points= $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

= $\sqrt{(7-4)^2 + (7-3)^2}$ = $\sqrt{3^2+4^2}$ = $\sqrt{25} = 5$ units

2. If the distance between the points (3, k) and (4, 1) is , then the value of k is

a. 4 or 2
b. –4
c. –4 or 2
d. 4 or –2

Sol: $\sqrt{(4-3)^2+(1-k)^2} = \sqrt{10}$

$\Rightarrow 1+1+k^2-2k = 10$

$\Rightarrow k^2-2k-8 = 0$

$\Rightarrow (k-4)(k+2) = 0$

⇒ k = 4 or k = –2

3. The point on the y-axis which is equidistant from A(–5, –2) and B(3, 2) is

a. (–4, 0)
b. (0, 2)
c. (0, –2)
d. (0, –4)

Sol: Let P(0, y) be the coordinates of the point on y-axis.

Then PA = PB ⇒ $\sqrt{(-5-0)^2+(-2-y)^2} = \sqrt{(3-0)^2+(2-y)^2}$

⇒ 25 + 4 + $y^2$ + 4y = 9 + 4 + $y^2$ – 4y

⇒ 8y = –16

⇒ y = -2

4. The vertices of a triangle are (–2, 0), (2, 3) and (1, –3), then the type of the triangle is

a. Right angle
b. Equilateral
c. Isosceles
d. Scalene

Sol: A(–2, 0), B(2, 3) and C(1, –3)

$AB = \sqrt{(2+2)^2+(3-0)^2} = 5$

$BC = \sqrt{(1-2)^2+(-3-3)^2} = \sqrt{37}$

$CA = \sqrt{(-2-1)^2+(0+3)^2} = 3\sqrt{2}$

$AB \neq BC \neq CA$ ⇒ Δ ABC is a scalene triangle

5. The points which are not collinear are

(I) (0, –1), (8, 3) and (6, 7)

(II) (4, 3), (5, 1) and (1, 9)

(III) (2, 5), (–1, 2) and (4, 7)

(IV) (–3, 2), (1, –2) and (9, –10)

a. I
b. II
c. III
d. IV

Sol: From option (a) A(0, –1), B(8, 3) and C(6, 7)

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$AB = \sqrt{(8-0)^2 + (3+1)^2} = 4\sqrt{5}$

$BC = \sqrt{(6-8)^2 + (7-3)^2} = 2\sqrt{5}$

$AC = \sqrt{(6-0)^2 + (7+1)^2} = 10$

$AB + BC \neq AC$

$\Rightarrow$ Points are not collinear.

6. The coordinates of the point which divides the line segment joining the points (–7, 4) and (–6, –5) internally in the ratio 7 : 2 is

a. $(\frac{56}{9}, 3)$

b. $(-\frac{56}{9}, -3)$

c. $(-\frac{61}{9}, 2)$

d. $(\frac{61}{9}, -2)$

Sol: Let (x, y) be the coordinates of the required point.

$x = \frac{7(-6)+2(-7)}{7+2} = \frac{-42-14}{9} = -\frac{56}{9}$

$y = \frac{7(-5)+2(4)}{7+2} = \frac{-35+8}{9} = -3$

$(x, y) \equiv (-\frac{56}{9}, -3)$

7. If the coordinates of the centroid of a triangle are (1, 3) and two of its vertices are (–7, 6) and (8, 5), then the third vertex of the triangle is

a. $(\frac{2}{3}, \frac{14}{3})$

b. $(-\frac{2}{3}, -\frac{14}{3})$

c. (2, –2)

d. (–2, 2)

Sol: Let (x, y) be the coordinates of the third vertex of the triangle. Then $\frac{-7+8+x}{3}$ = 1 and $\frac{6+5+y}{3}$ = 3 $\Rightarrow$ x + 1 = 3 and y + 11 = 9 $\Rightarrow$ x = 2 and y = –2 $\Rightarrow$ (x, y) = (2,

8. The ratio in which the line segment joining (3, 4) and (–2, –1) is divided by the x-axis is

a. 3 : 2

b. 1 : 4

c. 4 : 3

d. None of these

Sol: Let $\lambda$ : 1 be the required ratio. Then

$\frac{\lambda(-1)+1(4)}{\lambda+1} = 0$ [Ordinate of the point on the x-axis will be zero.]

$\Rightarrow \lambda$ = 4

The ratio is 4 : 1.

9. If a line passes through the mid-point of the line joining (–3, –4) and (–5, 6) and has a slope of $\frac{3}{4}$, then its equation is

a. 4x – 3y + 16 = 0

b. 3x – 4y + 16 = 0

c. 4x + 3y – 16 = 0

d. 3x + 4y – 16 = 0

Sol. Mid-points of (–3, –4) and (–5, 6) is $(\frac{-3-5}{2}, \frac{-4+6}{2})$

$[\text{Mid–point} \equiv (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})]$ i.e., (–4, 1)

The required equation of the line is $y - 1 = \frac{3}{4}(x+4)$ [Using $y - y_1 = m(x - x_1)$ ]

$\Rightarrow$ 3x – 4y + 16 = 0

10. The equation of the line through (2, –4) and parallel to the line joining the points (2, 3) and (–4, 5) is

a. 3x – y = 0

b. x + 3y = 0

c. x + 3y + 10 = 0

d. 3x – y = 14

Sol: Slope of the line joining (2, 3) and (–4, 5) is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{-4-2} = -\frac{1}{3}$

Slope of any line parallel to it = $-\frac{1}{3}$

The required equation of the line is y + 4 = $-\frac{1}{3}(x-2)$ [Using $y - y_1 = m(x - x_1)$ ]

$\Rightarrow$ x + 3y + 10 = 0

11. If the vertices of a triangle are (1, 3), (–2, 4) and (3, –5), then the equation of the altitude from (1, 3) to the opposite side is

a. 9x + 5y – 24 = 0

b. 9x – 5y + 24 = 0

c. 5x – 9y + 22 = 0

d. 5x + 9y – 22 = 0

Sol: Let the vertices be A(1, 3), B(–2, 4) and C(3, – 5).

Then the slope of BC =

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